# "Orthogonal functions"


$U_n(x),\ n = 1,2,3,4,\dots$

$$\int_a^b U_n^*(x)\, U_m(x)\, dx = \delta_{nm}$$

$$f(x) \approx \sum_{n=1}^{N} a_n\, U_n(x) \xrightarrow{MSE} a_n = \int_a^b U_n^*(x)\, f(x)\, dx$$

$$\boxed{\sum_{n=1}^{\infty} U_n^*(x)\, U_n(x') = \delta(x'-x)} \qquad \boxed{\nabla\left(\frac{1}{|x-x'|}\right) = -4\pi\, \delta(x-x')} \quad (I)$$

Eg. $$2\sum_{n=1}^{\infty} \sin(n\pi x)\, \sin(n\pi x') = \delta(x-x')$$

$$\delta(x) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega x}$$

$$2\sum_{n=0}^{\infty} \cos(n\pi x)\, \cos(n\pi x') = \delta(x-x')$$

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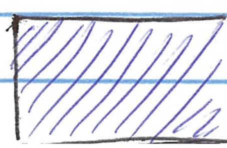
Sometimes we may assume that

$\Phi(x,y,z) = X(x)\, Y(y)\, Z(z)$ if so

$$\boxed{\frac{1}{X(x)}\frac{d^2X}{dx^2} = -\alpha^2 = \frac{-1}{Y(x)}\frac{d^2Y}{dy^2}}$$

$X = \Sigma \sin + \Sigma \cos$

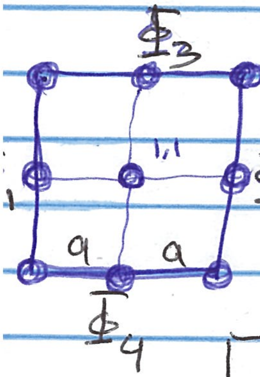
$y = \Sigma \sinh + \Sigma \cosh$



$X = \Sigma \sinh + \Sigma \cos$

$Y = \Sigma \sin + \Sigma \cos$

# Relaxation method




$$\Phi_2 \quad \Phi_{11} \simeq \frac{1}{4}\left(\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 - a^2 \nabla^2 \Phi_{11}\right)$$

$$\boxed{\Phi_{11,new} \simeq \frac{1}{4}\left(\Phi_{1,old} + \Phi_{2,old} + \Phi_{3,old} + \Phi_{4,old} - a^2 \rho_{11}\right)}$$

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$$\Phi(\rho,\phi) = a_0 + b_0 \ln\rho + \sum_{n=1}^{\infty}\left[a_n \rho^n \sin(n\phi + \alpha_n) + b_n \rho^{-n} \sin(n\phi + \beta_n)\right]$$

usuall we can write $a_n \sin(n\phi + \alpha_n) = c_n \cos(n\phi) + d_n \cos(n\phi)$

if we want to find potential inside cylinder

$b_0 = b_n = 0$

if we want to find potential outside cylinder

$a_n = 0$