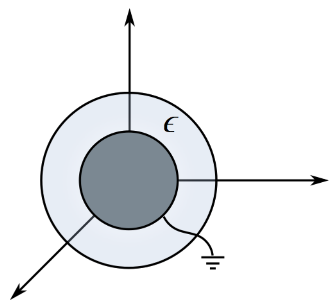
Practice Final Exam — II

The final will be a 180 minute open book, open notes exam. Do all four problems.

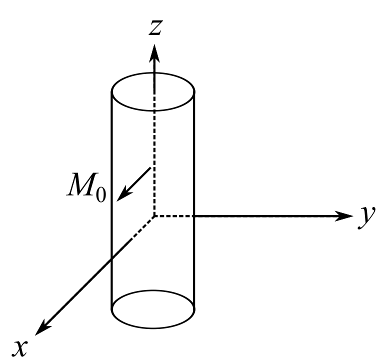
1. A grounded conducting sphere of radius a is surrounded by a dielectric medium of radius b (with $b > a$) and dielectric constant ϵ/ϵ_0. The combined system is placed in an initially uniform electric field, $\vec{E} = E_0\hat{z}$.



   (a) Find the electric potential everywhere.

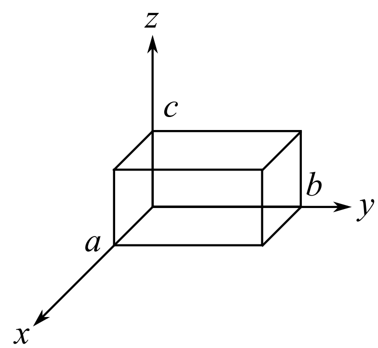
   (b) Show that the potential reduces to the expected forms in the limits $a = b$ (grounded conducting sphere) and $a = 0$ (dielectric sphere).

2. A magnetically "hard" material is in the shape of a right circular cylinder of length L and radius a. Take the limit $L \to \infty$, so the problem becomes effectively two-dimensional. The cylinder has a permanent magnetization M_0 uniform throughout its volume and *perpendicular* to its axis. To be concrete, take the cylinder axis to be the z-axis, and the magnetization to be pointed along the x-axis.



   Determine the magnetic field $\vec{H}$ and the magnetic induction $\vec{B}$ everywhere.

3. We wish to consider time-harmonic fields inside a hollow rectangular box of sides a, b and c.



The surfaces of the box are perfect conductors. Consider a “transverse magnetic” (TM) mode with the electric field given by $\vec{E} = E_z(x,y)\hat{z}e^{-i\omega t}$.

(a) Show that the electric field satisfies the Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2}\right) E_z(x,y) = 0$$

(b) Find the solution for $E_z(x,y)$ inside the box and show that only certain frequencies ω are allowed. Give an expression for these allowed frequencies.

4. An anisotropic but nonpermeable dielectric with electric displacement

$$D_x = \epsilon_1 E_x, \qquad D_y = \epsilon_2 E_y, \qquad D_z = \epsilon_3 E_z$$

fills the volume $z > 0$. A plane wave given by

$$\vec{E} = (E_1\hat{x} + E_2\hat{y})e^{i(kz-\omega t)} \qquad (k = \omega/c)$$

is normally incident on the dielectric from below.

(a) Show that the transmitted wave propagates with two different phase velocities, $v_1 = 1/\sqrt{\mu_0\epsilon_1}$ and $v_2 = 1/\sqrt{\mu_0\epsilon_2}$, for the $\hat{x}$ and $\hat{y}$ polarizations, respectively.

(b) Compute the power transmission and reflection coefficients

$$T = \frac{\text{transmitted power}}{\text{incident power}}, \qquad R = \frac{\text{reflected power}}{\text{incident power}}$$

and show that $T + R = 1$.